JOURNAL OF COMPUTATIONAL PHYSICS 5, 572–586 (1970)

Supersonic Flows About Conical Bodies

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Received January 9, 1970

The numerical solution of three problems of supersonic flow about conical bodies at zero and nonzero angles of attack is given.

The generalized method of integral relations is developed for calculating perfect gas flow about a cone at an angle of attack. The shock layer is subdivided into nonoverlapping strips by means of a number of rays and approximations by trigonometric polynomials with respect to the corresponding variable are carried out. The approximating system is integrated along these rays, starting from the shock wave, the coordinates of which are determined according to the condition of vanishing normal velocity on the body.

Supersonic flow about cones in the presence of an exothermal combustion reaction is analyzed. The two-component model is considered, in which the kinetics is described by a single concentration of unreacted molecules. The gas is assumed to be perfect with averaged thermodynamic properties, and direct and inverse reactions are taken into account after an induction delay time. In the general three-dimensional case the angular variable connected with the cross flow is eliminated from the governing system with the aid of trigonometric interpolations. The integration of the two-dimensional approximating system in all the meridian planes of interpolation is carried out by the numerical method of characteristics with a network of inverse type.

This characteristic computational scheme using two-dimensional compatibility relations is extended to the case of three-dimensional supersonic flows with nonequilibrium chemical processes, taking into account exact kinetics. The flow about blunt-nose inverted cones at an angle of attack in a supersonic stream of nonequilibrium dissociating oxygen is investigated.

INTRODUCTION

Conical surfaces are among the most widely used components for the construction of aerodynamic shapes flying at supersonic velocities. Hence the investigation of supersonic flows about conical bodies is a very important question, and it can be carried out effectively with the help of numerical methods. In this investigation we consider a number of different, stationary problems, namely such as the calculation of supersonic nonaxisymmetrical conical flow of a perfect gas, of the combustion in a supersonic stream flowing past conical bodies, and of the flow about blunt-nose inverted cones at angles of attack in the presence of nonequilibrium dissociation. In all these cases the gas is supposed to be inviscid and nonheat-conducting.

I. The Cone at Angles of Attack in a Supersonic Stream of a Perfect Gas

In calculating perfect gas flows about conical bodies at angles of attack, different authors have applied various numerical methods—the method of integral relations, the finite-difference method, and the method of straight lines in combination with the method of characteristics. The tables [1] of gasdynamic functions for circular cones at angles of attack have been computed by the finite-difference method. The first numerical solution of the direct problem of supersonic nonaxisymmetrical flow about conical bodies was obtained by the method of integral relations [2], where a scheme involving approximations across a shock layer was used. Another scheme of the method of integral relations involving approximations along a shock layer was proposed [3] (see also [4]). A practical realization of the last scheme is described here.

We confine ourselves to the case of flow with a plane of symmetry parallel to the free-stream velocity vector. A spherical polar coordinate system r, θ, ψ with the pole at the apex of the cone is used; the angle θ is measured from a fixed axis inside the body and the angle ψ from the windward side. Instead of θ we shall introduce a normalized variable

$$\xi = [\theta - \theta_B(\psi)]/[\theta_W(\psi) - \theta_B(\psi)],$$

where $\theta = \theta_B(\psi)$ and $\theta = \theta_W(\psi)$ are the equations of the traces of the body and shock wave on a sphere.

The system of equations of stationary conical flow of a perfect gas (see, for instance, [2]) is represented in general divergence form as follows

$$\frac{\partial P_i}{\partial \xi} + \frac{\partial Q_i}{\partial \psi} = \Omega_i , \tag{1}$$

where the subscript $i = 1, 2, 3, 4$ refers, respectively, to the momentum equations in the θ and ψ directions, to the continuity equation and to the entropy equation. Here P_i, Q_i, Ω_i are definite functions of the independent variables and of the basic gasdynamic functions—the components of the velocity vector $\bar{V}$, the pressure p, the density ρ and the entropy

$$S = (\gamma - 1)^{-1} \ln(p/\rho^\gamma),$$

where γ is the ratio of specific heats. Instead of the momentum equation in the r direction we shall use Bernoulli's integral.

The generalized method of integral relations will be applied to the solution of the problem. In the N-th approximation the region of integration $0 \leqslant \xi \leqslant 1, 0 \leqslant \psi \leqslant \pi$ is subdivided into N nonoverlapping strips with the help of rays

$$\psi = \psi_n = n\pi/N \;(n = 0, 1, \dots, N).$$

Keeping in mind the symmetry with respect to ψ, we multiply each of the equations (1) by $\sin n\psi$ or $\cos n\psi$. Then integrating with respect to ψ we form the following $4N$ integral relations

$$\frac{d}{d\xi}\int_0^\pi P_i \cos n\psi \, d\psi + n\int_0^\pi Q_i \sin n\psi \, d\psi = \int_0^\pi \Omega_i \cos n\psi \, d\psi,$$

$$i = 1, 3, \qquad n = 0, 1,..., N; \qquad i = 4, \qquad n = 0, 1,..., N-2,$$

$$\frac{d}{d\xi}\int_0^\pi P_2 \sin n\psi \, d\psi - n\int_0^\pi Q_2 \cos n\psi \, d\psi = \int_0^\pi \Omega_2 \sin n\psi \, d\psi,$$

$$n = 1, 2,..., N-1.$$

The odd functions $\mathscr{F}$ and even functions $\tilde{\mathscr{F}}$ in the integral relations are approximated by trigonometric polynomials with respect to ψ, which have interpolation nodes on all the rays $\psi = \psi_n$, i.e.,

$$\begin{aligned} \mathscr{F}(\xi, \psi) &= \sum_{n=1}^{N-1}\sum_{j=1}^{N-1} c_{nj}\mathscr{F}_j(\xi) \sin n\psi, \\ \tilde{\mathscr{F}}(\xi, \psi) &= \sum_{n=0}^{N}\sum_{j=0}^{N} d_{nj}\tilde{\mathscr{F}}_j(\xi) \cos n\psi, \end{aligned} \tag{2}$$

where c_{nj} and d_{nj} are numerical coefficients.

Using these integral relations we obtain an approximating system of ordinary differential equations in ξ, which can be solved with respect to the $4N$ values of the derivatives $dP_{in}/d\xi$ on all the rays $\psi = \psi_n$ in the form

$$\frac{dP_{in}}{d\xi} = \Omega_{in} + \sum_{j=0}^{N} K_{inj}Q_{ij}, \tag{3}$$

$$i = 1, 3, \qquad n = 0, 1,..., N; \qquad i = 2, 4, \qquad n = 1, 2,..., N-1,$$

where K_{inj} are numerical coefficients.

As basic unknown functions we shall take the values of the velocity component w in the ψ direction, the entropy S, the pressure p, the function P_3 on the rays $\psi = \psi_n (n = 0, 1,...N)$. The function P_3 is determined by the following expression

$$P_3 = \rho\left\{v \sin\theta - w\left[\xi\left(\frac{d\theta_W}{d\psi} - \frac{d\theta_B}{d\psi}\right) + \frac{d\theta_B}{d\psi}\right]\right\},$$

where v is the velocity component in the θ direction.

Taking into account the conditions of symmetry we have $w = 0$ on the rays $\psi = 0$ and $\psi = \pi$, and, in addition, the values of the entropy are constant on these rays and equal to their values immediately behind the shock wave. Therefore the total number of basic unknown functions will be $4N$, and the corresponding equations for the determination of the abovementioned unknown functions can be deduced from the $4N$ equations (3).

The resulting equations are integrated numerically with respect to ξ proceeding along the $N + 1$ rays $\psi = \psi_n$ from the shock wave ($\xi = 1$) to the body ($\xi = 0$). In the N–th approximation we must find those values of the angle θ_W on all the $N + 1$ rays, which satisfy the condition of vanishing normal velocity V_ν at the corresponding points on the cone surface. This condition is reduced to the equality $P_3 = \rho V_\nu = 0$. The equation of the shock wave is represented by the interpolation trigonometric polynomial in accordance with the assumed values θ_W. Then all the gasdynamic functions on the shock wave are found by known relations (see [2]).

It should be noted that on the cone surface the derivatives $dw/d\xi$ and $dS/d\xi$ as given by the approximating system become infinite, while the derivatives $dp/d\xi$ and $dP_3/d\xi$ have no singularity there. The same properties are obtained in the analytical investigation of the exact equations of conical flows.

To illustrate the above numerical scheme we shall present some calculated results concerning the circular cone $\theta_B = 20°$ in an air stream ($\gamma = 1.4$) at Mach number $M_\infty = 7$ and various angles of attack, $\alpha = 5°, 10°, 15°$. The numerical

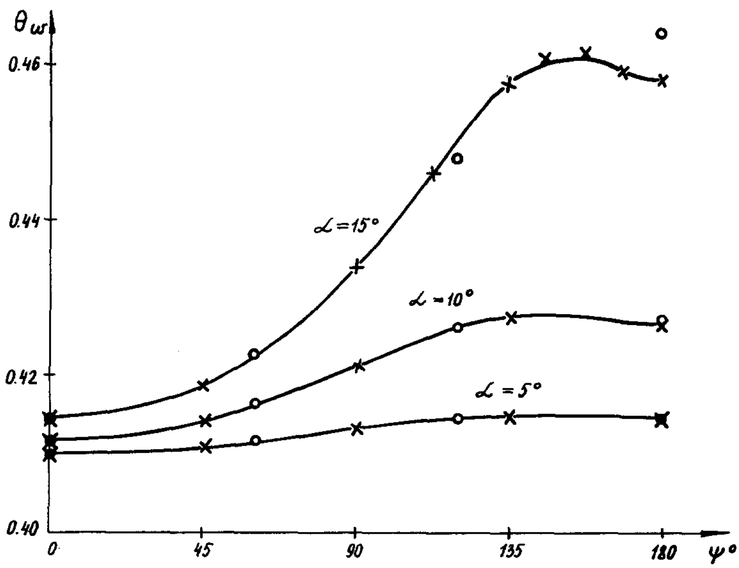

FIG. 1. Shape of shock wave for cone at angles of attack in supersonic stream of perfect gas.

solution was computed using different approximations in the method of integral relations. The results are shown in the graphs by circles for $N = 3$ and by solid lines for $N = 4$. For comparison data from the tables [1] as calculated by a finite-difference method are also displayed here (marked by crosses in the graphs).

Fig. 1, where the curve $\theta = \theta_W(\psi)$ is plotted, gives the form of the shock wave. It can be seen from this graph that one sensitive detail of the flow—the maximum of this curve near the leeward side of the body—was already determined with sufficiently good accuracy in the approximation $N = 4$. Fig. 2 shows how the dimensionless pressure (referred to $\rho_\infty a_{cr}^2$, where ρ_∞ is the free-stream density, a_{cr} is the critical velocity of sound), changes along the cone surface. The results presented demonstrate that in the cases calculated, practical accuracy is achieved for $N = 4$.

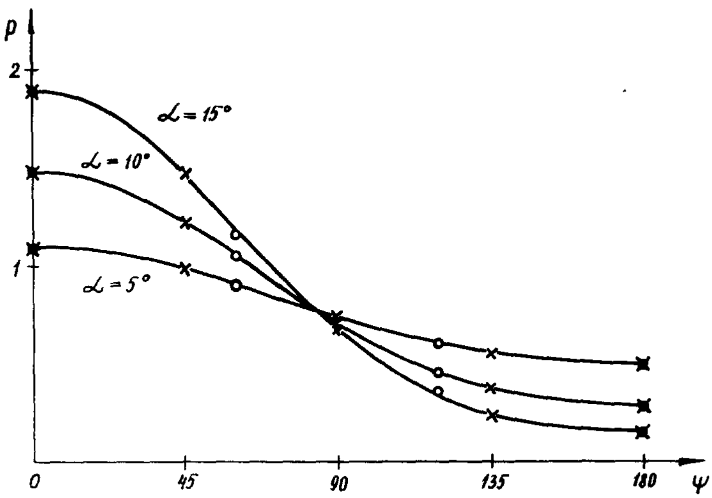

FIG. 2. Pressure distribution on cone at angles of attack in supersonic stream of perfect gas.

II. THE COMBUSTION IN A SUPERSONIC STREAM PAST A CONE

For stationary supersonic flow about conical bodies in the presence of non-equilibrium combustion reactions, the problem becomes nonselfsimilar and depends on two or three space variables in the cases of or and nonzero angles of attack, respectively. We now consider the problem of calculation of supersonic flow of a combustible gas about a body when an exothermal reaction takes place behind the shock wave.

As is well-known, the process of combustion for hydrogen-air mixtures consists of two stages. At first, during the period characterized by an induction delay time, the reactions proceed without any heat release. Subsequently, the reactions have a thermal effect. For a hydrogen-air mixture it is advisable to introduce some simplified models of the combustion kinetics, which prove to be quite reliable.

We shall assume a two-component model in which the initial mixture and the products of its combustion are considered, but the kinetics of the exothermal reaction are described by a single variable c—the mass concentration of unreacted molecules. The gas is supposed to be perfect, but it has averaged thermodynamic properties (the ratio of specific heats γ and the molecular weight η) depending on c. The investigation of combustion in supersonic flows about different bodies has been carried out on the basis of such a model [5, 6].

In this case the system of gas dynamics equations will be

$$\nabla \cdot \rho \bar{V} = 0, \qquad \rho(\bar{V} \cdot \nabla)\, \bar{V} + \nabla p = 0, \qquad \rho \bar{V} \cdot \nabla h - \bar{V} \cdot \nabla p = 0. \tag{4}$$

The equation of state and the expression for the enthalpy h should be added to (4), namely

$$p = \frac{R}{\mu} \rho T, \qquad h = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + qc, \tag{5}$$

where R is the universal gas constant, T is the temperature, q is the heat release per unit mass of combustible mixture.

The concentration c will be found from a kinetic equation taking into account simultaneously the direct reaction (combustion) and the inverse reaction (recombination), having the following general form

$$\frac{dc}{dt} = -k_1 c^m p^l \exp\left(-\frac{E}{RT}\right) + k_2 (1 - c)^m\, p^n \exp\left(-\frac{E + q}{RT}\right). \tag{6}$$

Here k_1 and k_2 are constants of the reaction rate, E is the activation energy, m is the order of the reaction, l and n are exponents.

The induction delay time can be expressed by the formula

$$t_{\text{ind}} = \frac{k}{p^{m-1}} \exp\left(-\frac{E_1}{RT}\right), \tag{7}$$

where k and E_1 are positive constants. We shall introduce the fraction of induction delay time φ, to be described by the following equation

$$\frac{d\varphi}{dt} = \frac{1}{t_{\text{ind}}}. \tag{8}$$

In the zone of induction $0 \leqslant \varphi \leqslant 1$ (it is obvious that on the boundary of ignition $\varphi = 1$) we have $c = 1$, $q = 0$ and the flow is adiabatic.

The governing system of Eqs. (4)–(8) in the general three-dimensional case will be given in cylindrical coordinates x, r, ψ, with the body given by the equation $r = r_B(x, \psi)$. The shock wave is an adiabatic discontinuity surface for which the well-known Rankine–Hugoniot relations hold and where the concentration $c = 1$.

Using the normalized variable $\xi = (r - r_B)/(r_W - r_B)$, we shall compute the solution on successive layers which are perpendicular to the body axis. Each layer is represented as a plane ψ = const with fixed nodal points formed by the intersection of a series of surfaces ξ = const and ψ = const. We shall eliminate the derivatives with respect to ψ from the governing system of equations in the variables ψ, ξ, ψ by applying the trigonometric approximations of type (2). The resulting approximating system of differential equations in the variables ψ and ξ defines the values of the basic unknown functions on all the meridian planes of interpolation ψ = const.

Under certain supersonic conditions, this approximating system is hyperbolic, possessing in each meridian plane two families of characteristics and one family of lines, analogues to stream lines. Therefore, the numerical method of characteristics may be applied to the integration of this approximating system. Here we use an implicit computational scheme of second-order accuracy, developed in [7–10], based on the projection of characteristics from the nodal points on the layer to be calculated, towards the previous, known layer. The computational algorithms of

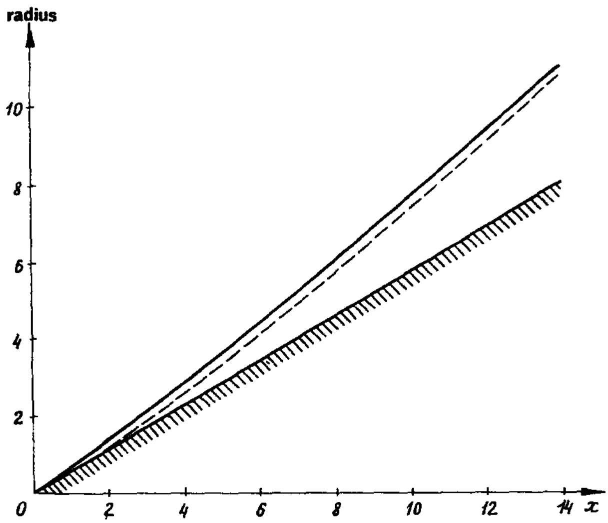

FIG. 3. Shock wave and ignition boundary for combustion in supersonic flow about cone at zero angle of attack.

this scheme include iterations and interpolations. This scheme is valid, naturally, both in the particular case of axisymmetric flow and in the general case of three-dimensional flow about nonconical bodies.

The calculations of supersonic flow about cones in the presence of an exothermal combustion reaction have been carried out for a stoichiometric hydrogen-air mixture. In Figs. 3–6 are shown some numerical results for a circular cone with semi-apex angle $\theta_B = 30°$. The free-stream has the following parameters—the Mach number $M_\infty = 5$, the angle of attack $\alpha = 0°$, the temperature $T_\infty = 685°K$, the pressure $p_\infty = 1$ atm. All the data are given in dimensionless form, assuming as reference quantities the free-stream density ρ_∞, the critical velocity of sound $a_{cr} = 1380$ m/sec and the induction delay time $t_{ind} = 0.52 \times 10^{-6}$ sec.

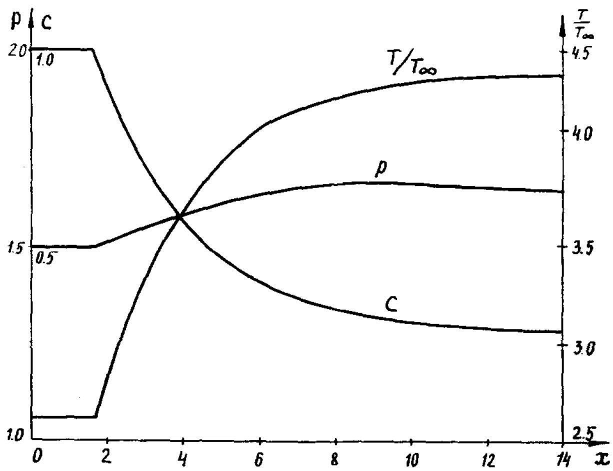

FIG. 4. Distributions of concentration, temperature, and pressure on cone for combustion in supersonic stream at zero angle of attack.

The shock wave (solid line) and the boundary of ignition (dashed line) are plotted in Fig. 3. In the process of combustion the shock wave increases its inclination, thus the pressure and the temperature behind it also increase and the induction delay time diminishes. As a result, the boundary of ignition approaches the shock wave; in this case the normal velocity component behind the shock wave continues to be subsonic.

The distribution of physical parameters—the concentration c, the temperature T/T_∞ and the pressure p—along the cone surface are shown in Fig. 4. Immediately after the induction period, the development of the combustion is very intense, being accompanied by a significant increase in temperature, and subsequently the process of combustion slows down. The concentration c on the cone surface decreases quickly, tending to a value at which the direct and inverse reactions are in equilibrium. The pressure along the body increases at first, reaching an insignificant maximum. In the computations, some oscillations depending on the activation energy were observed in the flow at increasing ψ.

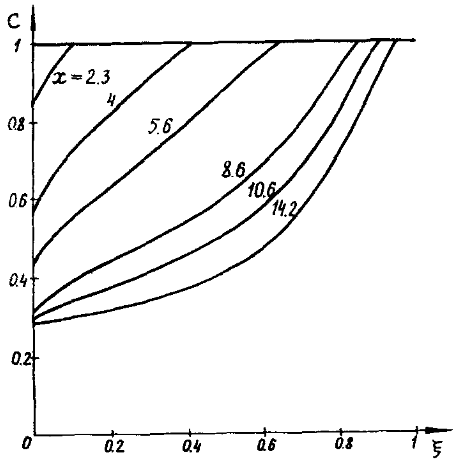

FIG. 5. Concentration distribution across shock layer for combustion in supersonic flow about cone at zero angle of attack.

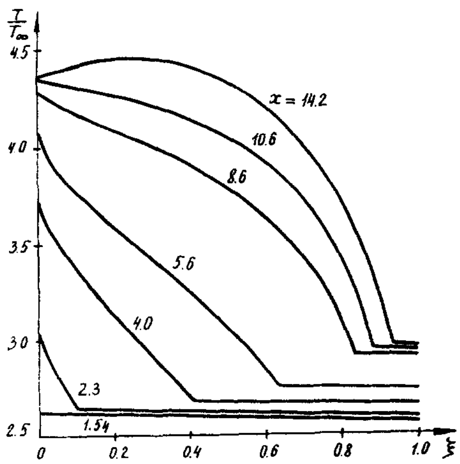

FIG. 6. Temperature distribution across shock layer for combustion in supersonic flow about cone at zero angle of attack.

The structure of the combustion zone is illustrated by Figs. 5 and 6, where the distributions of mass concentration and of temperature between the cone surface and the shock wave are presented for a series values of ψ. When crossing the boundary of ignition the flow parameters have discontinuities of their derivatives. Equilibrium is established at first near the body surface, while near the shock wave there is nonequilibrium flow. It is interesting that the maximum temperature before equilibrium occurs on the body, and later on the maximum temperature is inside the flow-field. It should be noted that the pressure reaches its maximum value at the boundary of ignition behind which a rarefaction wave is present.

The numerical analysis of combustion of a hydrogen-air mixture in the flow about a cone at angles of attack was also carried out. The effect of heat release on three-dimensional flow, in principle, is the same as in the case of zero angle of attack. However, the process of combustion on the windward side ($\psi = 0°$), where the pressure and the temperature are higher, proceeds more intensively than on the leeward side ($\psi = 180°$). In this case the maximum schock layer thickness in a plane $x =$ const takes place inside the region $0° < \psi < 180°$.

For illustration, some numerical results are presented for the cone with the semiapex angle $\theta_B = 30°$, and for the following free-stream parameters: the angle of attack $\alpha = 10°$, the Mach number $M_\infty = 7$, the temperature $T_\infty = 400°K$, and the pressure $p = 1$ atm. The surface distributions of concentration (solid line) and of temperature (dashed line) versus the variable ψ are shown in Fig. 7. The surface distributions of the same functions versus the variable ψ are drawn in Figs. 8 and 9.

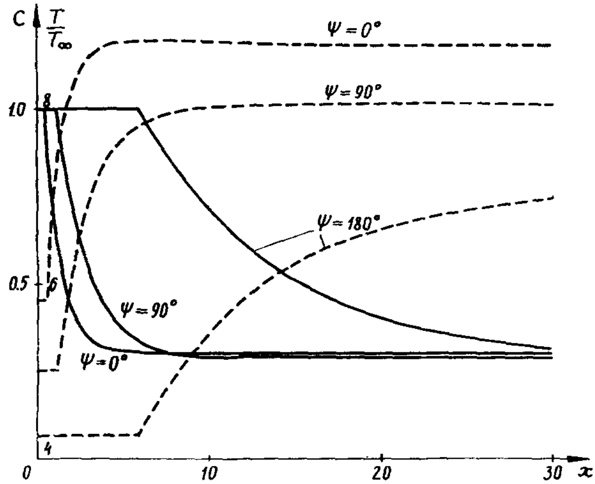

FIG. 7. Distribution of concentration and temperature on cone for combustion in supersonic flow at angle of attack.

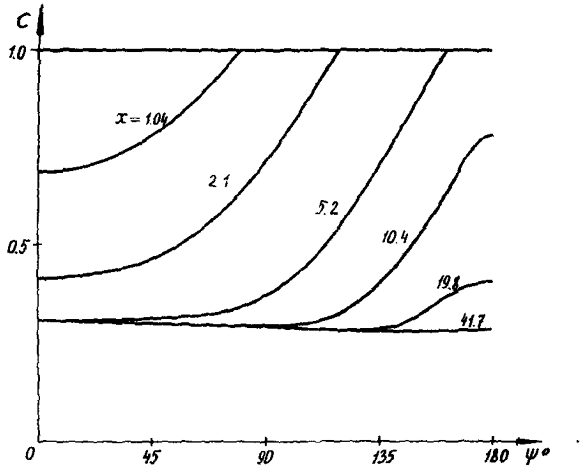

FIG. 8. Concentration distribution on cone for combustion in supersonic flow at angle of attack.

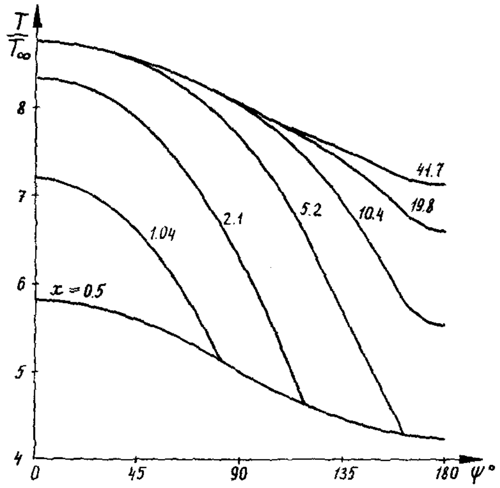

FIG. 9. Temperature distribution on cone for combustion in supersonic flow at angle of attack.

III. THE BLUNT-NOSE INVERTED CONE IN A SUPERSONIC STREAM OF NONEQUILIBRIUM DISSOCIATING OXYGEN

The above model of combustion kinetics permits us to clear up the principal features and the distributions of physical parameters in nonequilibrium streams. However, it is interesting to calculate supersonic two-dimensional and three-dimensional flows of a nonperfect gas, taking into account the exact kinetics involving reactions with finite rates.

For such a problem Eqs. (4) hold again, but now the equation of state and the expression for the enthalpy must be considered in the general form

$$\rho = \rho(p, T, c_1, ..., c_m), \qquad h = h(p, T, c_1, ..., c_m), \tag{9}$$

where c_i is the mass concentration of the i-th component of the gas medium ($i = 1, 2, ..., m$), m is the number of components.

The rate of change of concentration of the i-th component due to all the l chemical reactions will be expressed by the following general equation

$$\frac{dc_i}{dt} = \sum_{j=1}^{l} \varphi_{ij}(p, T, c_1, ..., c_m) f_{ij}(p, T, c_1, ..., c_m). \tag{10}$$

Here the function φ_{ij} is proportional to the rate of the j-th chemical reaction. The concrete forms of the expressions (9) and (10) are given by chemical kinetics.

In order to solve the problem of nonequilibrium supersonic gas flow, the governing system (4), (9), (10) is integrated numerically with the aid of the characteristic computational scheme described in the preceding section and developed in detail in [8–10].

As we approach equilibrium conditions $\varphi_{ij} \to \infty$ and $f_{ij} \to 0$ in the kinetic equations. This fact can lead to an instability in the numerical solution. Usually computational schemes of an implicit type are applied to avoid these difficulties. A special implicit computational scheme of second-order accuracy was worked out in [8–10] for the stable numerical integration of kinetic equations (10) in the case of three-dimensional supersonic flows close to equilibrium conditions. In this scheme the functions f_{ij} are not computed explicitly at equilibrium conditions, but are represented by a two-term expansion in that parameter c_i, which tends to the equilibrium value. In order to calculate this parameter a finite-difference formula is derived which ensures stability of the computations both near to, and far from, equilibrium.

The given numerical method has been applied in [11] for the solution of the problem of supersonic three-dimensional flow about a blunt-nose inverted cone set at an angle of attack in a stream of oxygen in the presence of nonequilibrium dissociation. We have considered the case when the internal degrees of freedom were in a state of equilibrium, and there was no ionization. The kinetic equations in these computations were assumed to be the same as those in [8].

The calculations were carried out for bodies which have as the nose part a sphere with radius $r_B = 1$ m and as the aft part a circular inverted cone with a semi-apex angle θ_B. We present some numerical results for the case when the undisturbed nondissociating stream has a pressure $p_\infty = 0.001$ atm, a temperature $T_\infty = 288°$K and a Mach number $M_\infty = 10$ (corresponding to the free-stream velocity $V_\infty = 3236$ m/sec). All the functions in the following graphs are dimensionless; the radius r_B, the density ρ_∞, the velocity V_∞ and the gas constant of nondissociating oxygen are taken as the reference quantities.

The numerical solution has been obtained using nine meridian planes of interpolation in the region $0° \leqslant \psi \leqslant 180°$. In addition, the solution with five planes of interpolation has been calculated—the corresponding data are shown by crosses in the graphs and they are in good agreement with the basic solution.

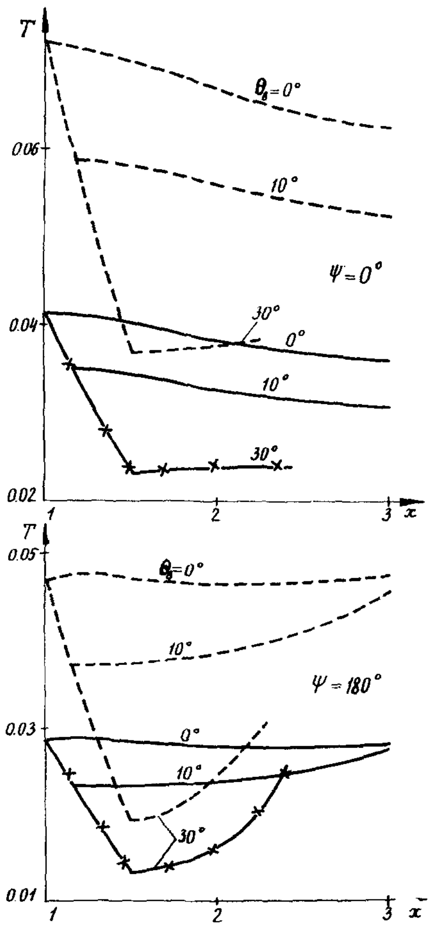

FIG. 10. Temperature distribution on blunt-nose inverted cones at angle of attack in supersonic stream of nonequilibrium dissociating oxygen.

For comparison, the flows of a perfect diatomic gas have been calculated for the same blunt-nose inverted cones at the Mach number $M_\infty = 10$. The analysis proves that the nonequilibrium dissociation essentially affects the temperature distribution, and has little influence on the pressure distribution. In Fig. 10 the temperature distribution along the windward ($\psi = 0°$) and leeward ($\psi = 180°$) body generators are given for two blunt-nose inverted cones $\theta_B = 10°$ and $\theta_B = 30°$, and for a blunt-nose cylinder $\theta_B = 0°$ at angle of attack $\alpha = 10°$. The solid lines correspond to the flow with nonequilibrium dissociation and the dashed lines correspond to the flow of a perfect gas. The right-hand parts of these curves are related to the conical surface of the body. In the vicinity of the aft point of the body there is a strong retardation of the flow accompanied by an increase in temperature and pressure.

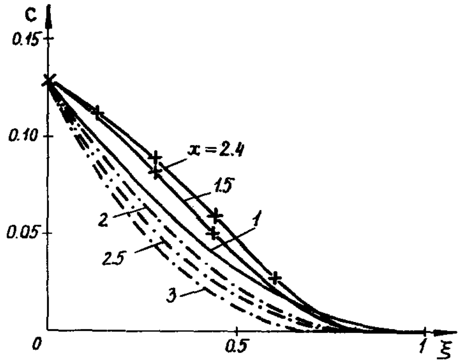

FIG. 11. Concentration distribution across shock layer on blunt-nose inverted cones at angle of attack in supersonic stream of nonequilibrium dissociating oxygen.

Fig. 11 shows how the mass concentration of atomic oxygen c varies across the shock layer, depending on the variable ξ for a series of values of ψ and $\psi = 0°$. These data are drawn for inverted cones with semiapex angles $\theta_B = 10°$ (dash-dotted line) and $\theta_B = 30°$ (solid line) at angle of attack $\alpha = 10°$. One can see that the concentration proves to be practically frozen on the whole conical surface of the body ($\xi = 0$).

Finally, the graphs of the coefficient of normal force C_n and of the coefficient of longitudinal moment C_m, referred to the dynamic head and the maximum cross-section area of the body, are plotted in Fig. 12 for the angles of attack $\alpha = 10°$ (solid line) and $\alpha = 15°$ (dashed line). For comparison, the analogous data for the same bodies in the case of a perfect gas are depicted by circles and triangles. It is clear that a nonequilibrium dissociation influences the aerodynamic coefficients very little, since its effect upon the pressure distribution on the body surface is small.

The above calculations of nonequilibrium three-dimensional supersonic flows maintain the computational efficiency of the scheme proposed even in the case of bodies with a curvature discontinuity and large gradients of gasdynamic functions.

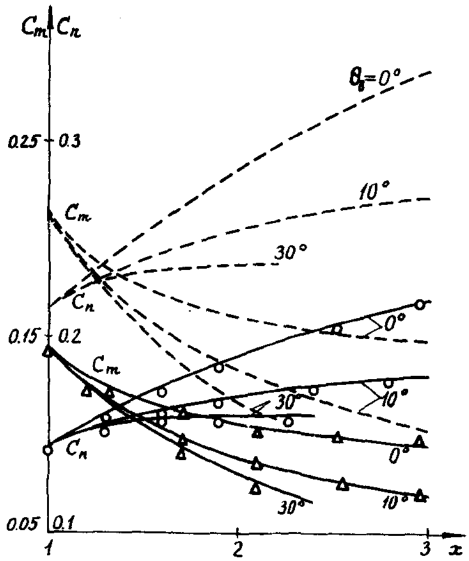

FIG. 12. Coefficient of normal force and coefficient of longitudinal moment for blunt-nose inverted cones at angle of attack in supersonic stream of nonequilibrium dissociating oxygen.

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